Two-Dimensional Wind-Tunnel Interference from Measurements on Two Contours

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This paper describes how wall induced velocities near a model in a two dimensional wind tunnel can be estimated from upwash distributions measured along two contours surrounding a model The method is applicable to flows that can be represented by linear theory It was derived by applying the Schwarz Integral Formula separately to the two contours and by exploiting the free air relationship between upwashes along the contours Advantages of the method are that only one flow quantity need be measured and no representation of the model is required A weakness of the method is that it assumes streamwise interference velocity vanishes far upstream of the model This method was applied to a simple theoretical model of flow in a solid wall wind tunnel The theoretical interference velocities and the velocities computed using the method were in excellent agreement The method was then used to analyze experimental data acquired during adaptive wall experiments at Ames Research Center This analysis confirmed that the wall adjustments reduced wall induced velocities near the model

Nomenclature

c = airfoil chord cm
H = tunnel height cm
M = Mach number
u = streamwise velocity perturbation m/s
U = freestream velocity m/s
w = vertical velocity perturbation (upwash) m/s
x = longitudinal coordinate positive downstream from model cm
y = vertical coordinate positive above model cm
α = angle of attack deg
β = $(1-M^2)^{1/2}$
Γ = circulation m^2/s
μ = source strength m^3/s

Subscripts

f = field level
i = wall induced
m = model induced
s = source level
o = centerline

Introduction

IN the past decade several new methods have been developed for correcting wind tunnel data for wall interference These methods use measurements of one or more flow quantities on a contour surrounding the wind tunnel model to estimate wall induced flow perturbations near the model They eliminate much of the empiricism and uncertainty characteristic of earlier correction methods; however making the required flow measurements can be a formidable task

J Smith identified three types of new wall correction methods applicable to two dimensional flows [1] Methods of the first type require that one flow quantity be measured on a contour surrounding the model In addition flow perturbations due to the model in free air must be estimated along the contour One additional condition must be imposed to evaluate a constant of integration Smith called these 'Schwarz'' methods because they can be derived from the Schwarz Integral Formula [2]

Methods of the second type require that two flow quantities be measured on the control contour Smith called these ''Cauchy ' methods because they can be derived from the Cauchy Residue Theorem Although Cauchy methods can pose a more challenging flow measurement problem than Schwarz methods they eliminate any need to estimate model induced flow perturbations and there is no constant of integration to evaluate

Cauchy methods are most easily applied in solid wall test sections where the walls can serve as the control contour Velocities normal to the walls are zero (neglecting the wall boundary layer) and velocities parallel to the walls can be determined from static pressure measurements

Both Cauchy and Schwarz methods assume that the flow in the wind tunnel can be described by linear equations Measured perturbations are assumed to be the superposition of perturbations resulting from the model in free air (at some 'corrected angle of attack and Mach number) and perturbations produced by the tunnel walls This occurs if wall induced velocities are uniform near the wall Test conditions become increasingly uncorrectable as wall induced velocity gradients near the model grow

The third type of correction method is not limited to linear flows Wall corrections are determined by matching the pressure distribution measured on the model with that computed for an effective model in free air at a corrected angle of attack and Mach number These methods require pressure measurements on the model and on a contour surrounding the model Flow about the model must be calculated both in free air and with pressure boundary conditions imposed at the control contour Smith calls these 'matching methods

This paper describes a method for computing wall inducd velocities along the centerline of a two dimensional wind tunnel from upwash distributions measured along two contours surrounding the model This two level method is an adjunct to adaptive wall procedures developed at Ames It allows corrections for wall interference to be computed directly from the same measurements used to determine wall adjustments

The two level method combines advantages of the Schwarz and Cauchy methods As with Schwarz methods only one flow quantity (upwash) need be measured and as with Cauchy methods there is no need to estimate perturbations resulting from the model A disadvantage of the method is that measurements must be made along two contours rather than one Also as with Schwarz methods a constant of in

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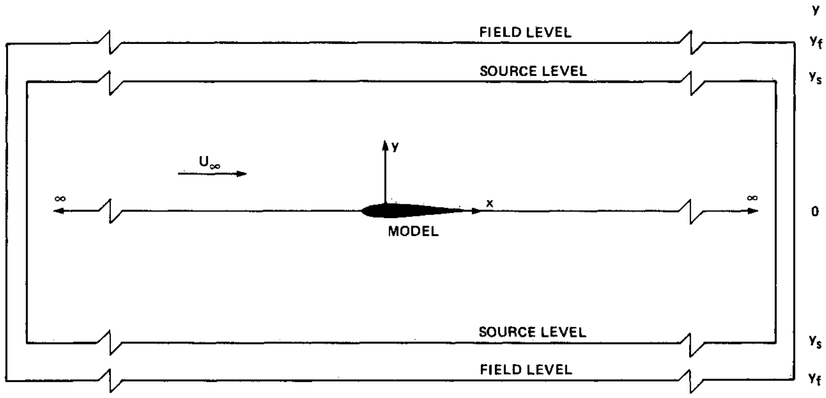


Fig 1 Geometry for two level correction method

tegration must be evaluated Like both Schwarz and Cauchy methods the present method is limited to linear flows

This paper presents a derivation of the two level correction method shows how the method was tested by applying it to a simple potential flow and presents an analysis of experimental data previously acquired in the Ames 25 × 13 cm adaptive wall test section

The Two-Level Method

The two level method was derived by applying the Schwarz formula separately to two contours surrounding the model and by exploiting the free air relationship between upwash distributions along the two contours Vertical and streamwise velocity distributions induced by the tunnel walls along the centerline of the tunnel are expressed in terms of the upwash distributions measured along the two contours

Consider the geometry illustrated in Fig 1 The model located at $y=0$ is surrounded by two contours Each contour consists of two horizontal lines equidistant above and below the model aligned with the freestream and extending upstream and downstream to infinity The contours are closed by vertical legs at their upstream and downstream to infinity The contours are closed by vertical legs at their upstream and downstream extremities; however these legs may be ignored since they are at infinity We call the horizontal lines closest to the model "source" levels and the outer lines field" levels

Assume that the pressure distribution on the model is identical to that which would occur in free air at some corrected angle of attack and Mach number Flow perturbations in the test section are assumed to be the sum of those due to the model at the corrected free air conditions and those due to the tunnel walls The problem is to determine the wall induced perturbations along the tunnel centerline from upwash distributions measured along the source and field levels

Let

$$\bar{w}(x\ y) = \frac{w(x\ y) + w(x\ -y)}{2}$$

be the symmetric component of w With the Schwarz formula the wall induced upwash along $y=0$ (w_{o_i}) can be determined from the symmetric component of the wall induced upwashes at level y

$$w_{o_i}(x\ 0) = \frac{1}{\beta y}\int_{-\infty}^{\infty} \frac{\bar{w}_i(\xi\ y)}{2\cosh(\pi/2\beta y)(x-\xi)}\,\mathrm{d}\xi \qquad (1)$$

The symmetric components of the model induced upwashes at the source ($\bar{w}_{s_m}$) and field ($\bar{w}_{f_m}$) levels are uniquely related by a linear, free air transformation[3]

$$\bar{w}_{f_m}(x\ y_f) = \frac{\beta\,|y_f - y_s|}{\pi}\int_{-\infty}^{\infty} \frac{\bar{w}_{s_m}(\xi\ y_s)}{(\xi-x)^2 + \beta^2(y_f-y_s)^2}\,\mathrm{d}\xi \quad (2)$$

Let operators A_s and A_f represent the Schwarz formula [Eq (1)] at the source ($y=y_s$) and field ($y=y_f$) levels respectively Likewise let E represent the free air transformation [Eq (2)] Then

$$w_{o_i} = A_f \bar{w}_{f_i} \qquad (3)$$

$$w_{o_i} = A_s \bar{w}_{s_i} \qquad (4)$$

$$\bar{w}_{f_m} = E\bar{w}_{s_m} \qquad (5)$$

Model induced upwashes can be expressed as differences between total upwashes and wall induced perturbations

$$\bar{w}_{f_m} = \bar{w}_f - \bar{w}_{f_i} \qquad \bar{w}_{s_m} = \bar{w}_s - \bar{w}_{s_i}$$

Substituting these expressions into Eq (5) and rearranging terms yields

$$\bar{w}_f - E\bar{w}_s = \bar{w}_{f_i} - E\bar{w}_{s_i} \qquad (6)$$

We next apply the Schwarz operator A_f to both sides of Eq (6) and eliminate $\bar{w}_{f_i}$ using Eqs (3) and (4)

$$A_f(\bar{w}_f - E\bar{w}_s) = (A_s - A_f E)\,\bar{w}_{s_i}$$

Since $\bar{w}_f$ and $\bar{w}_s$ are measured and thus known we can solve for $\bar{w}_{s_i}$

$$\bar{w}_{s_i} = (A_s - A_f E)^{-1}\ A_f(\bar{w}_f - E\bar{w}_s) \qquad (7)$$

Finally, w_{o_i} is found from Eq (4) This is not the most direct derivation but it requires that only one operator be inverted

The streamwise velocity distribution induced by the walls along $y=0$ (u_{o_i}) can be derived in a similar manner from the antisymmetric components of the upwash distributions measured at the source and field levels Let

$$\tilde{w}(x\ y) = \frac{w(x\ y) - (w\ -y)}{2}$$

be the antisymmetric component of w Then by the Schwarz formula

$$u_{o_i}(x\ 0) = \frac{-1}{\beta^2 y}\int_{-\infty}^{\infty} \frac{\tilde{w}_i(\xi\ y)}{e^{-(\pi/\beta y)(x-\xi)} + 1}\,\mathrm{d}\xi \qquad (8)$$

Implicit in Eq (8) is a constant of integration which was determined by assuming that u_{o_i} vanishes at upstream infinity

Let operator B represent the linear transformation in Eq (8) The free air operator E applies to the antisymmetric as well as the symmetric components of w_m Thus

$$u_{o_i} = B_f \tilde{w}_{f_i} \qquad (9)$$

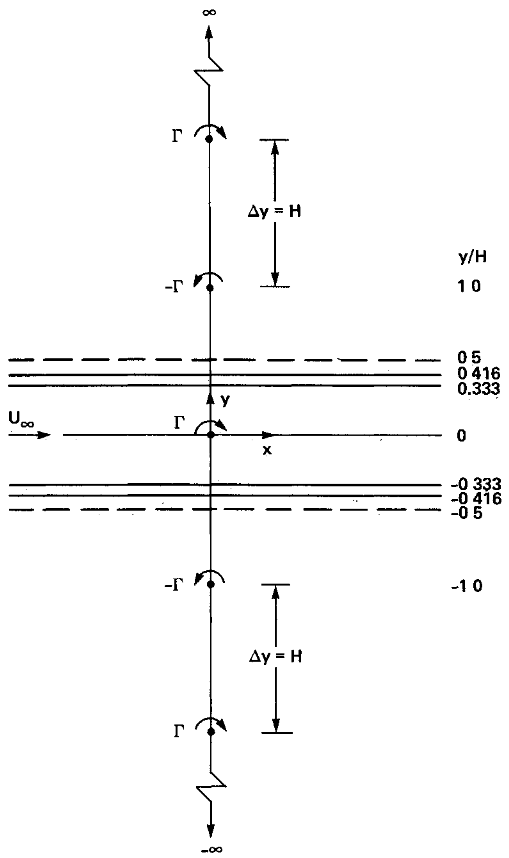


Fig 2 Theoretical model of flow in a solid wall wind tunnel

$$u_{o_i} = B_s \tilde{w}_{s_i} \tag{10}$$

$$\tilde{w}_{f_m} = E\tilde{w}_{s_m} \tag{11}$$

If $\tilde{w}_{s_i}$ is derived in terms of the operators B_s B_f and E the operator that must be inverted $(B_s - B_f E)$ is ill conditioned This occurs because the influence function $(e^{-(\pi/\beta y)(x-\xi)}+1)^{-1}$ in Eq (8) does not approach zero for large values of $(x-\xi)$ This problem can be avoided by taking the derivative of Eq (8) with respect to x[4 5]

$$u'_{o_i}(x\ 0) = \frac{du_{o_i}}{dx}$$

$$= \frac{-\pi}{2\beta^3 y^2} \int_{-\infty}^{\infty} \frac{\tilde{w}_i(\xi\ y)}{\cosh(\pi/\beta y)(x-\xi)+1} d\xi \tag{12}$$

Let B represent this transformation Then

$$u_{o_i} = B'_f \tilde{w}_{f_i} \qquad u_{o_i} = B_s \tilde{w}_{s_i}$$

The same steps used to derive Eq (7) then yield

$$\tilde{w}_{s_i} = (B_s - B'_f E)^{-1}\ B'_f(\tilde{w}_f - E\tilde{w}_s) \tag{13}$$

Eq (10) can then be used to find u_{oi}

Note that the terms $(\tilde{w}_f - E\tilde{w}_s)$ in Eq (7) and $(\tilde{w}_f - E\tilde{w}_s)$ in Eq (13) are the symmetric and antisymmetric components of the error function used in the two level adaptive wall algorithm to determine wall adjustments

Method of Solution

Equations (1) and (8) were derived by adapting the ap proach described by J Smith [4] Whereas Smith s derivation was in terms of the wall induced streamwise velocities along a control contour Eqs (1) and (8) are expressed in terms of wall induced upwashes

In the application of the present method Eqs (1) (2) (8) and (12) were solved by numerical integration Continuous upwash distributions were approximated by sets of upwashes at discrete points and integrals were replaced by sums The sums were truncated upstream and downstream of the model where upwashes were assumed to be very small

As an example of the numerical method consider the approximation to Eq (1) at the point $(x_j\ 0)$:

$$w_{o_i}(x_j\ 0) = \sum_{k=1}^{n} a_{jk}\tilde{w}_i(x_k\ y) \tag{14}$$

where

$$a_{jk} = \frac{1}{\beta y} \frac{(\Delta x_k/2)}{2\cosh(\pi/2\beta y)(x_j - x_k)}$$

$$\Delta x_k = x_{k+1} - x_{k-1} \quad \text{for } 1<k<n$$

$$= x_{k+1} - x_k \quad \text{for } k=1$$

$$= x_k - x_{k-1} \quad \text{for } k=n$$

Eq (14) was used to approximate w_{o_i} at n points along $y=0$ and the influence coefficients a_{jk} were assembled into a square $n \times n$ matrix A Matrices E B and B were determined in a similar manner from Eqs (2) (8) and (12) respectively Operator arithmetic was perfomed according to the rules of linear algebra

Test Case

Flow within a solid wall wind tunnel was simulated by a simple arrangement of singularities (Fig 2) The model was represented by the superposition of a doublet and a point vortex at the origin Images of the model were spaced at regular intervals (H) along the y axis The strength of each image was equal to that of the model but the circulations were of alternating sign

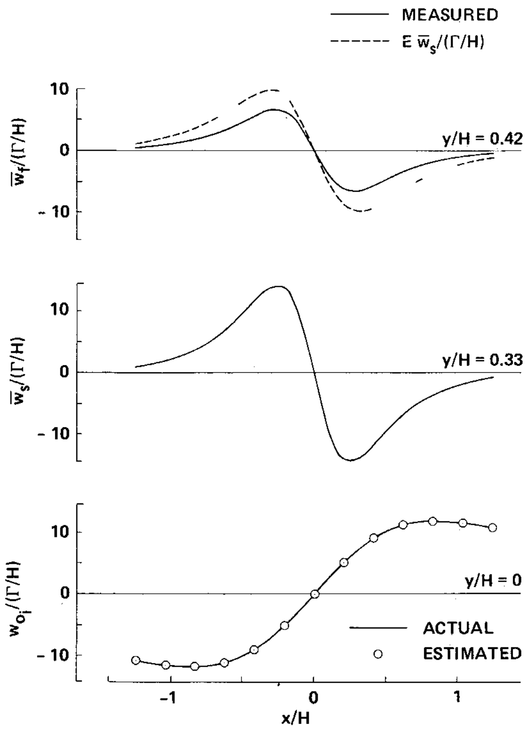
Streamlines at $y/H = \pm 0\ 5$ represented the tunnel walls The wall induced velocity at any point was the sum of the velocities due to all the images Measured'' velocities were the sums of velocities due to the model and walls

This conceptual wind tunnel was used as a test case for the two level method Source and field levels were defined at $y/H = \pm 0\ 33$ and $\pm 0\ 416$ respectively Measured upwash distributions at these levels were separated into symmetric (Fig 3a) and antisymmetric (Fig. 3b) components due to the vortices and doublets respectively Figure 3 also includes the symmetric and antisymmetric components of the function (Ew_s)

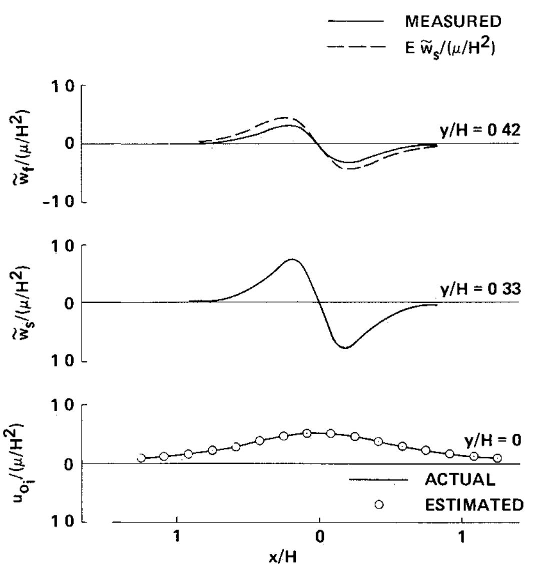
Wall induced streamwise velocities (u_{o_i}) along the tunnel centerline were estimated from Eqs (13) and (10) and wall induced upwashes (w_{o_i}) were estimated from Eqs (7) and (4) The actual wall perturbations along $y=0$ are illustrated in Fig 3 To the scale of the figure the perturbations estimated by the two level method were indistinguishable from the actual perturbations

Analysis of Data from an Adaptive-Wall Experiment

Two dimensional airfoil experiments in the Ames adaptive wall wind tunnel were completed in 1980 [6] An NACA 0012 airfoil was tested at $\alpha = 0$ and 2 deg and at $M = 0\ 6$ to 0 8 The chord to height ratio of the model in the test section was 0 60 The top and bottom walls of the test section were slotted Separate upper and lower plenums were divided into com partments in which the pressures could be adjusted Upwash distributions were measured with a laser velocimeter at two levels above and two levels below the airfoil, and wall ad justments were determined from Davis' two level com patibility assessment method [3]



a) Symmetric



b) Antisymmetric

Fig 3 Upwash distributions at the source and field levels and wall induced upwash along the tunnel centerline

In what follows two sets of data from these experiments are analyzed by the two level correction method The effect of wall adjustments on the wall induced perturbations along the tunnel centerline are estimated

Figure 4 illustrates where upwashes were measured in the adaptive wall test section At the source levels ($y/c = \pm 0\ 40$) upwashes at the most upstream and downstream measurement stations (a and d Fig 4) were nearly zero At the field levels ($y/c = \pm 0\ 67$) however measurements were not made as far upstream and downstream and upwashes at the extreme points (stations b and c) were generally not zero Therefore upwash distributions at the field levels were ex trapolated by assuming that the error function ($w_f - Ew_s$) varied linearly upstream from station b and downstream from station c, to zero at stations a and d respectively Beyond points a and d upwashes at the source and field levels were assumed to decay in inverse proportion to the distance from the model Upwash distributions were interpolated between measurement points using Aitken's scheme [7]

Figure 5 illustrates upwash distributions at the source and field levels for the case $\alpha = 2\ 0$ deg, $M = 0\ 65$ Two sets of data are shown One (Fig 5a) was acquired before the test section walls were adjusted and the other (Fig 5b) after The function (Ew_s) is also illustrated

Figure 6 shows how the distributions of u_{o_i} and w_{o_i} estimated by the two level method were affected by the wall adjustments Before they were adjusted the walls produced a downwash angle of about 0 063 rad (3 6 deg) at $x/c = 0\ 5$ and accelerated the flow along the centerline Gradients of the perturbations were quite large near the model

The large gradients illustrated in Fig 6 suggest that this initial condition is not correctable by the two level (or any similar) method This is also suggested by the fact that the lift on the model was positive whereas the corrected angle of attack [assumed to be the sum of the geometric angle of the model and the wall induced flow angle at $x/c = 0\ 5$ (Ref 8)] was negative

After the walls were adjusted both the vertical and streamwise wall perturbations were much smaller At $x/c = 0\ 5$ the estimated wall induced downwash angle was 0 012 rad (0 68 deg) and u_{o_i} was reduced from about 5.5 to 0 7% of the freestream velocity Gradients of u_{o_i} and w_{o_i} near the model were also much smaller after the walls were ad justed These residual corrections were substantial however and additional wall adjustments would have been required to eliminate interference

Figure 7 illustrates upwash data from the second example acquired at $\alpha = 0$ deg and $M = 0\ 78$ The upwash field was assumed to be antisymmetric above and below the model and upwashes were measured only at the upper source and field levels The figure includes data acquired before (Fig 7a) and after (Fig 7b) the walls were adjusted

For an antisymmetric upwash field w_{o_i} would be zero Thus only distributions of u_{o_i} were estimated (Fig 8) Initially the walls accelerated the flow upstream of the model

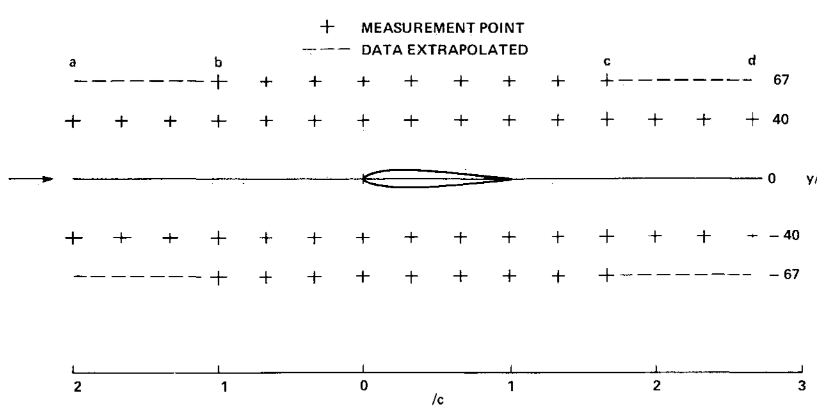
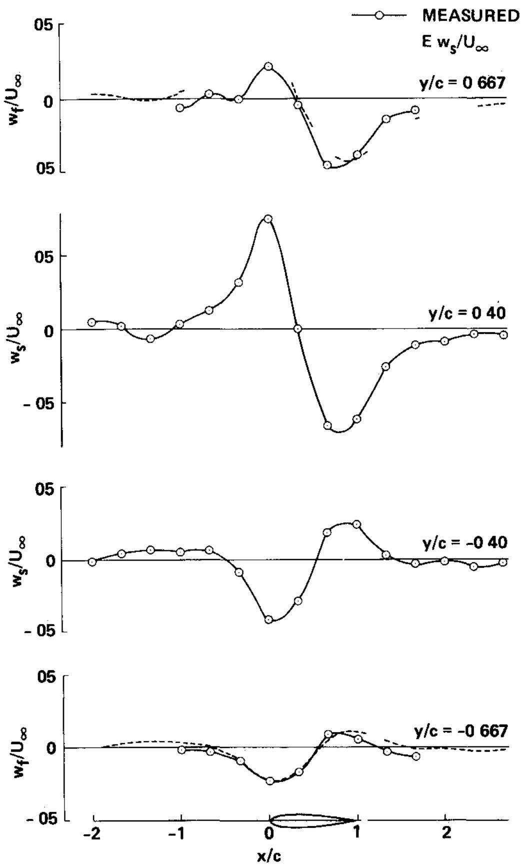
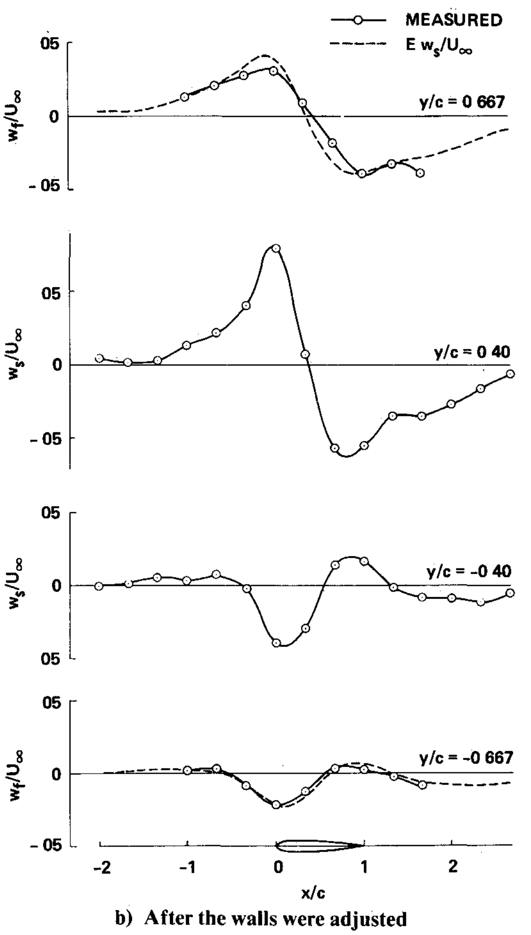


Fig 4 Locations where upwashes were measured in the adaptive wall experiment



a) Before the walls were adjusted



b) After the walls were adjusted

Fig 5 Upwashes measured at the source and field levels ($\alpha = 2$ 0 deg $M = 0$ 65)

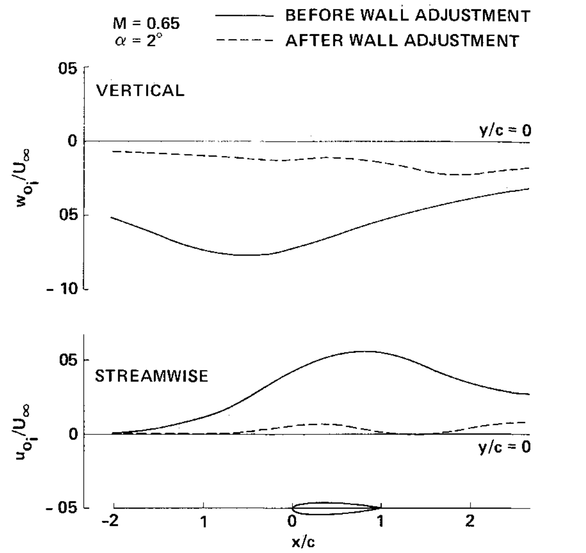
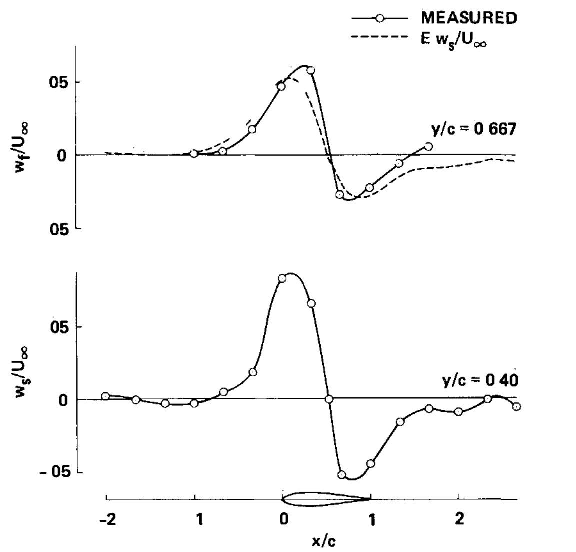
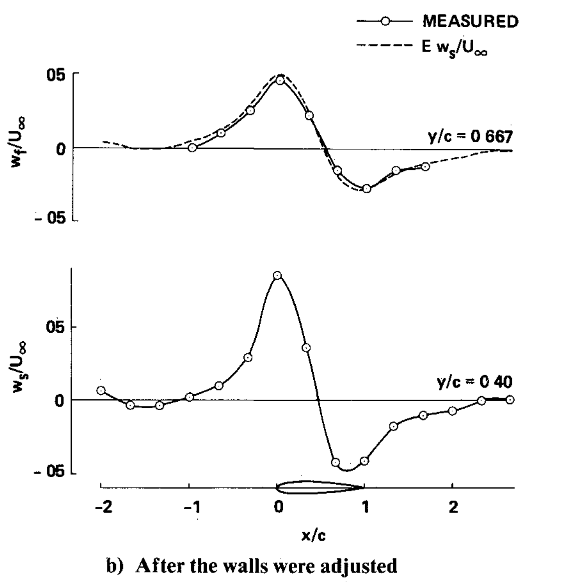


Fig 6 Estimated wall induced velocities along the tunnel centerline ($\alpha = 2$ 0 deg $M = 0$ 65)



a) Before the walls were adjusted



b) After the walls were adjusted

Fig 7 Upwashes measured at the source and field levels ($\alpha = 0$ deg $M = 0$ 78).

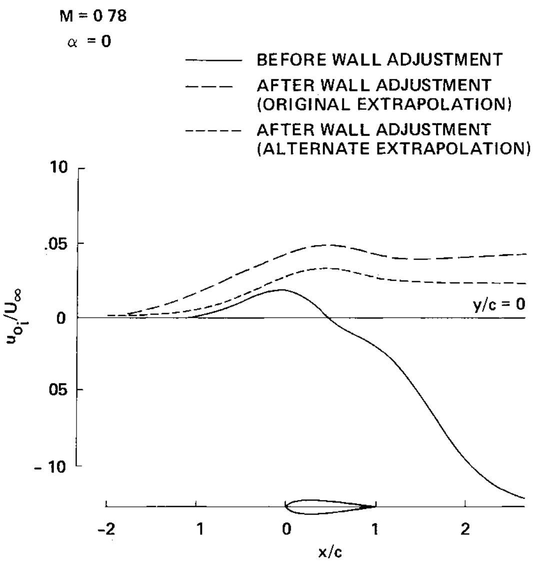


Fig 8 Estimated wall induced streamwise velocities along the tunnel centerline ($\alpha = 0$ deg $M = 0$ 78)

and decelerated the flow downstream of the model Although u_{oi} was zero at $x/c = 0$ 5 the gradient near the model was appreciable Wall adjustments actually increased u_{oi} near the model but decreased the streamwise gradients of $u_o i$

At this second test condition supersonic zones existed near the upper and lower surfaces of the airfoil thus violating the assumption that flow in the wind tunnel can be described by linear equations Although the shock waves were weak the two level method was not strictly applicable in this case

Discussion

The accuracy of the two level method depends upon how accurately the error function $(w_f - Ew_s)$ is known The error function and by extension sensitivity to interference is greatest if the separation between the source and field levels is a maximum i e if the field levels are at the walls and the source levels are at the model This optimum is not possible in practice since the source level must lie between the model and the tunnel walls The field levels could be located at the walls of a solid wall wind tunnel ($w_f = 0$, neglecting the wall boundary layer) In a test section with ventilated walls however the field levels must be removed from the walls because the flow near the wall is generally viscous and three dimensional and the velocity normal to the wall cannot be easily determined

In the theoretical test case the separation between the source and field levels was far from optimum This did not affect the accuracy of the estimated interference velocities because upwashes at the source and field levels were measured with infinite precision

In the adaptive wall experiments placement of the source and field levels was constrained by experimental con siderations The source levels were located just above and below an optical obstruction produced by the model supports The field levels were located as close to the slotted walls as possible $[y_f/(H/2) = \pm 0$ 78] without being in the wall boundary layers or in the regions where three dimensional disturbances due to the slots were important

Upwashes in the adaptive wall experiments were measured with sufficient accuracy (about 0 005 U_∞) to resolve the error function $(w_f - Ew_s)$ before the walls were adjusted (Figs 5a and 7a) After the walls were adjusted the error function at many of the measurement points was smaller than could be resolved (Figs 5b and 7b)

In principle the error function must be known upstream and downstream to infinity The test section in the test case was infinitely long and ' measurements' extended far upstream and downstream of the model where the error function was nearly zero Thus trunaction errors were small In addition the condition that u_{o_i} vanish far upstream was exactly satisfied in this idealized test section

The range of upwash measurements at the field levels in the adaptive wall experiments was very limited and the error function had to be extrapolated This resulted in significant uncertainty in the estimates of the wall induced velocities For example u_{o_i} was recomputed from the data in Fig 7b with the assumption that the error was zero upstream of station b and downstream of station c (The original assumption was that the error decayed to zero linearly between station b and a and stations c and d) This small difference in assumption resulted in a substantial reduction in the estimated magnitude of u_{o_i} near the model (Fig 8)

Another source of uncertainty in determining u_{o_i} was the assumption that $u_{o_i} = 0$ far upstream of the model In the adaptive wall experiments the model was located five chord lengths downstream of the test section inlet The axial Mach number distribution was quite constant several chord lengths upstream of the model This supports the assumption that $u_{o_i} = 0$ far upstream of the model

Concluding Remarks

A method has been developed for estimating wall induced velocities near a two dimensional wind tunnel model from upwashes measured along two contours surrounding the model Advantages of the method are: 1) only upwash needs to be measured and 2) no representation of the model is necessary Weaknesses of the method are: 1) upwashes must be measured on two contours 2) placement of the contours for maximum sensitivity to interference is often impractical and 3) wall induced streamwise velocities are assumed to vanish far upstream of the model

The method accurately predicted the wall induced velocities along the centerline of a theoretical solid wall wind tunnel The method s weaknesses were of little importance since there was no experimental uncertainty in the upwash 'measurements and the assumption that u_{o_i} vanishes at upstream infinity was appropriate

Analysis of experimental data acquired during adaptive wall tests confirmed that wall adjustments substantially reduced wall interference There was however significant uncertainty in the estimated interference This was due to the limited range over which upwashes were measured measurement in accuracies and nonoptimum placement of the control contours

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